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How we "look" inside stars: stellar evolution codes & MESA

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"Traditional scientific knowledge has generally taken the form of either theory or experimental data. However, where theory and experiment stumble, simulations may offer a third way."
Simulation, Johannes Lenhard *et al.*

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The most important thing

What is (Computational) “Stellar Astrophysics”?

The MESA stellar evolution code

- Basic Assumptions
- Discretization
- Translation of the Physics for the Computer
- Example of input Physics: Nuclear Reaction Networks
- How the Computer Solves the Equations

What do I do with it?

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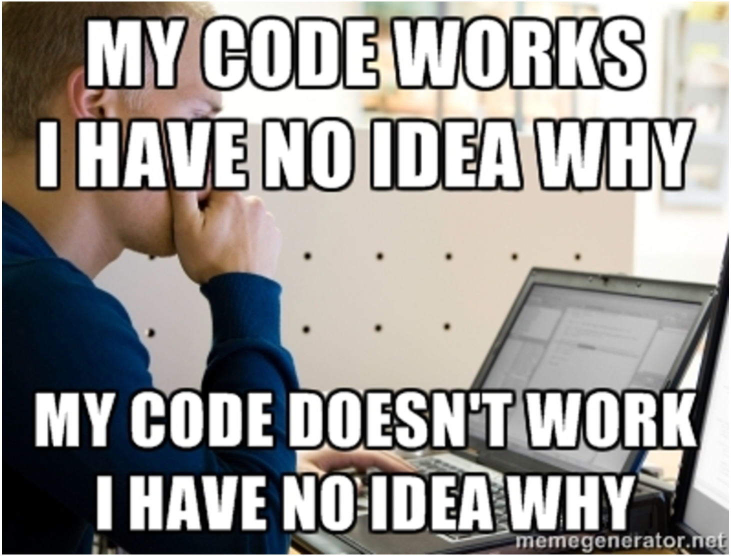
This is what should **not** happen



`grep` is your friend! (see `man grep` on *nix)

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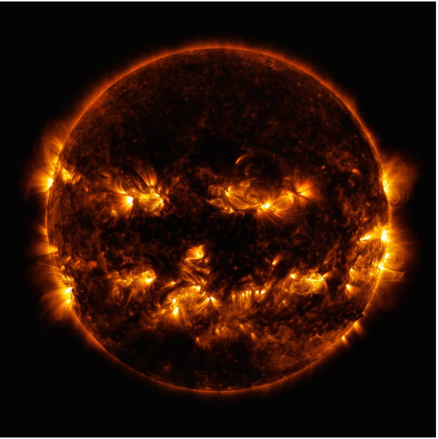
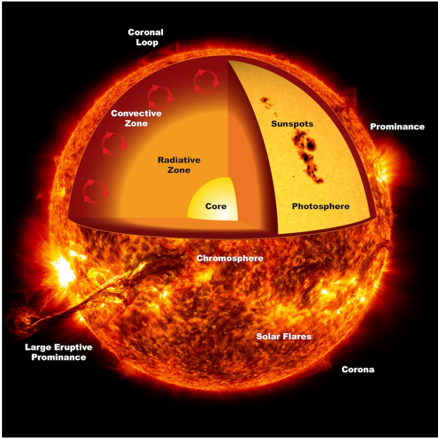
Figures Credits: NASA



$\stackrel{?}{\Longrightarrow}$



How can we "look" inside a star?

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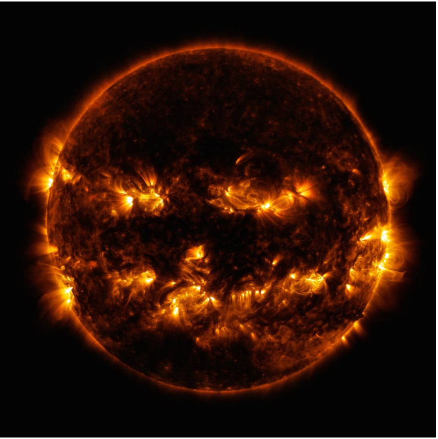
Figures Credits: NASA



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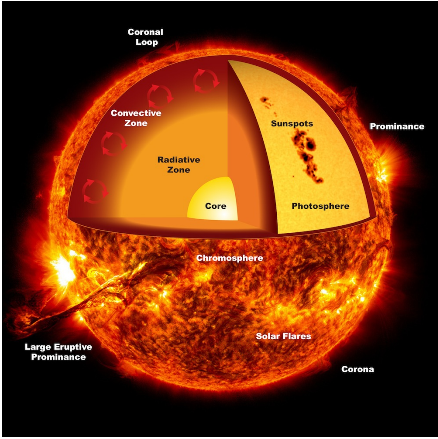

We simply can't!!

Other Q: How can we observe how *one* star evolves?

So what to do?

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1. Build a theory from first principles;
2. Plug it in a computer;
3. Get out a ***model***;
4. Find a smart way to compare it to what we *can* observe.

Advantages

- Full control over the parameters $\Rightarrow$ Numerical Experiments;
- Allow to focus on interesting things (e.g. no reddening!);
- Allow to deal with long-lasting, rare, inaccessible phenomena;

Drawbacks

- Numerical errors;
- Limited computational resources;
- Nature $\gg$ Theory $\gg$ Model.

“All models are wrong, but some are useful” – G. Box

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MESA

is a *tool*, not a theory!

What does it stand for?

Modules for Experiments in Stellar Astrophysics

References:

Paxton *et al.* 2011, ApJs192,3
Paxton *et al.* 2013, ApJs208,4
Paxton *et al.* 2015, ApJs220,15
mesa.sourceforge.net
mesastar.org

Open Source ⇔ Open Know How

"An algorithm must be seen to be believed" – D. Knuth

How to get MESA:

svn co -r 7624 svn://svn.code.sf.net/p/mesa/code/trunk mesa

Modules overview

MESA Module Definitions and Purposes

Name	Type	Purpose
alert	Utility	Error handling
atm	Microphysics	Gray and non-gray atmospheres; tables and integration
const	Utility	Numerical and physical constants
chem	Microphysics	Properties of elements and isotopes
diffusion	Macrophysics	Gravitational settling and chemical and thermal diffusion
eos	Microphysics	Equation of state
interp_1d	Numerics	One-dimensional interpolation routines
interp_2d	Numerics	Two-dimensional interpolation routines
ionization	Microphysics	Average ionic charges for diffusion
jina	Macrophysics	Large nuclear reaction nets using reaclib
kap	Microphysics	Opacities
karo	Microphysics	Alternative low-T opacities for C and N enhanced material
mlt	Macrophysics	Mixing length theory
mtx	Numerics	Linear algebra matrix solvers
net	Macrophysics	Small nuclear reaction nets optimized for performance
neu	Microphysics	Thermal neutrino rates
num	Numerics	Solvers for ordinary differential and differential-algebraic equations
package_template	Utility	Template for creating a new MESA module
rates	Microphysics	Nuclear reaction rates
screen	Microphysics	Nuclear reaction screening
star	Evolution	One-dimensional stellar evolution
utils	Utility	Miscellaneous utilities
weaklib	Microphysics	Rates for weak nuclear reactions

Prohibitive computational cost of 3D simulations

$\Rightarrow$ 1D, but stars are *not* spherical-symmetric!

Need of parametric approximations for:

- Rotation $\Rightarrow$ “Shellular Approximation”;
- Magnetic Fields;
- Convection $\Rightarrow$ Mixing Length Theory (MLT);
- (Some) mixing processes;
- ...

Beware of systematic errors!

(Recall: Nature $\gg$ Model)

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$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2}$$

... but stars are not necessarily static!

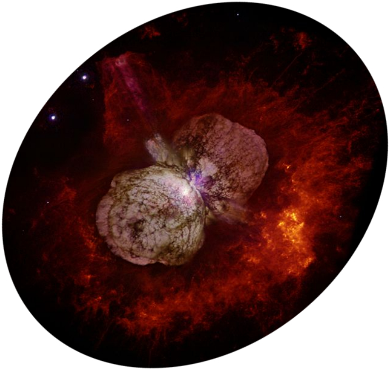

Figure: η Car, APOD.

Other examples:

- He flash,
- Outburst and Eruptions,
- Impulsive mass loss,
- RLOF,
- ...

For numerical solutions:

$$\frac{df}{dx} \rightarrow \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k}$$

$\Rightarrow$ Discretization of space (mesh or grid) and time (timesteps)

(Recall: Nature $\gg$ Model)

Spatial Discretization (Meshing)

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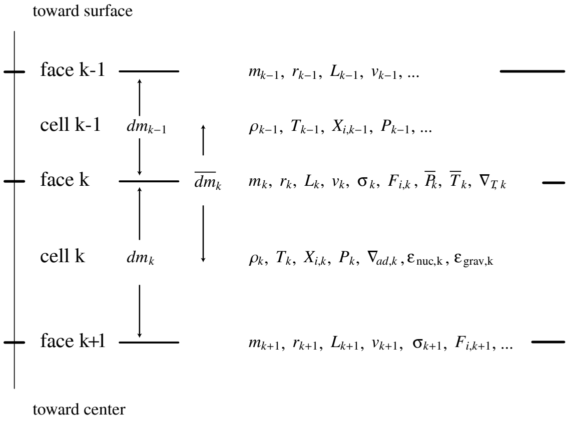

- Intensive quantities (e.g. T,ρ) averaged by mass within each cell;
- Extensive quantities (e.g. m,L) calculated at outer cell boundary.

Figure: From Paxton *et al.* 2011, ApJs, 192, 3

Need to check that your physical results do *not* depend on the way you discretize space.

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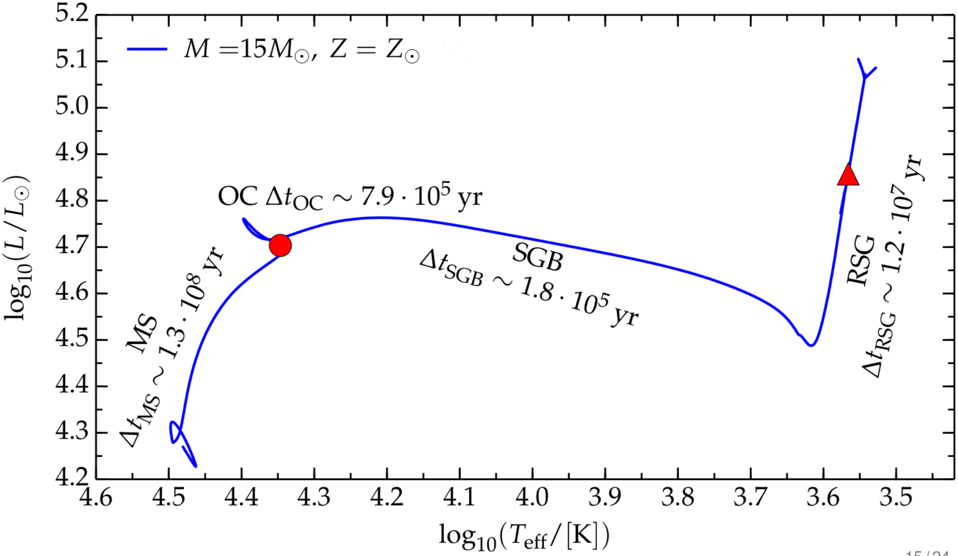
Δt_n: Large enough, but $\lesssim \tau_{\rm KH}, \tau_{\dot{M}}$, etc.

Need to find the best Δt_n at each step – few$\times$100$\lesssim$total n $\lesssim$few$\times 10^4$



Reformulation of the (1D–) Equations

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Physical Theory: Numerical Implementation:

$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2} \left(+ \frac{F}{4\pi r^2} \right) \Leftrightarrow$$

$$\frac{dm}{dr} = 4\pi r^2 \rho \Leftrightarrow$$

$$\frac{dT}{dr} = -\frac{3}{16\pi ac} \frac{\kappa\rho L}{r^2 T^3} \Leftrightarrow$$

$$\frac{dL}{dr} = 4\pi r^2 \rho\varepsilon \Leftrightarrow$$

$$P \equiv P(\rho, \mu, T) \Leftrightarrow$$

$$\left.\frac{dX_i}{dt}\right|_r = \left[\sum_j \mathcal{P}_{j,i}(T, \rho) - \sum_k \mathcal{D}_{i,k}(T, rho,) \right] + \left[\sigma_i \nabla^2 X_i \right]$$

$$\Updownarrow$$

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$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2} \left(+\frac{F}{4\pi r^2}\right) \Leftrightarrow \frac{P_{k-1}-P_k}{0.5(dm_{k-1}-dm_k)} = -\frac{Gm_k}{4\pi r_k^4} - \frac{a_k}{4\pi r_k^2}$$

$$\frac{dm}{dr} = 4\pi r^2\rho \Leftrightarrow \ln(r_k) = \frac{1}{3}\ln\left[r_{k+1}^3 + \frac{3}{4\pi}\frac{dm_k}{\rho_k}\right]$$

$$\frac{dT}{dr} = -\frac{3}{16\pi ac}\frac{\kappa\rho L}{r^2 T^3} \Leftrightarrow \frac{T_{k-1}-T_k}{(dm_{k-1}-dm_k)/2} = -\nabla_{T,k}\left(\left.\frac{dP}{dm}\right|_k\right)\frac{\tilde{T}_k}{\tilde{P}_k}$$

$$\frac{dL}{dr} = 4\pi r^2\rho\varepsilon \Leftrightarrow L_k - L_{k+1} = dm_k\{\varepsilon_{\rm nuc} - \varepsilon_\nu + \varepsilon_{\rm grav}\}$$

$$P \equiv P(\rho,\mu,T) \Leftrightarrow P \equiv P(\rho,\mu,T)$$

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$$\Updownarrow$$

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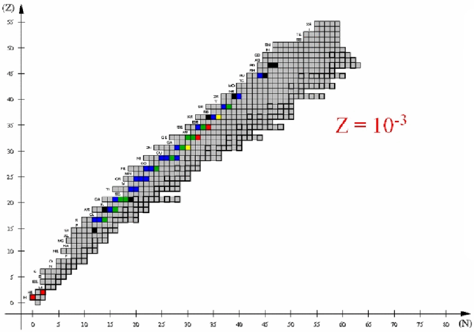

What matters:

- Total Number of Isotopes $N_{\rm iso}$;
- Which Isotopes;
- Number of Nuclear Reactions.

(Ex. of) tricks under the hood:

- Compound reactions, e.g. 3α: $\alpha + \alpha \rightarrow \left(^{8}\mathrm{Be} + \alpha \rightarrow\right) {}^{12}\mathrm{C} + \gamma$;
- (Quasi Statistical Equilibrium Networks for advanced burning stages);

High impact on:

- Computational cost ($\propto N_{\rm iso}^2$) $\Rightarrow$ Run time;
- $\varepsilon_{\rm nuc} \Rightarrow L$, T_c, ρ_c, etc.;
- Free electrons (Y_e) $\Rightarrow$ Final fate (BH,NS, WD, etc.)

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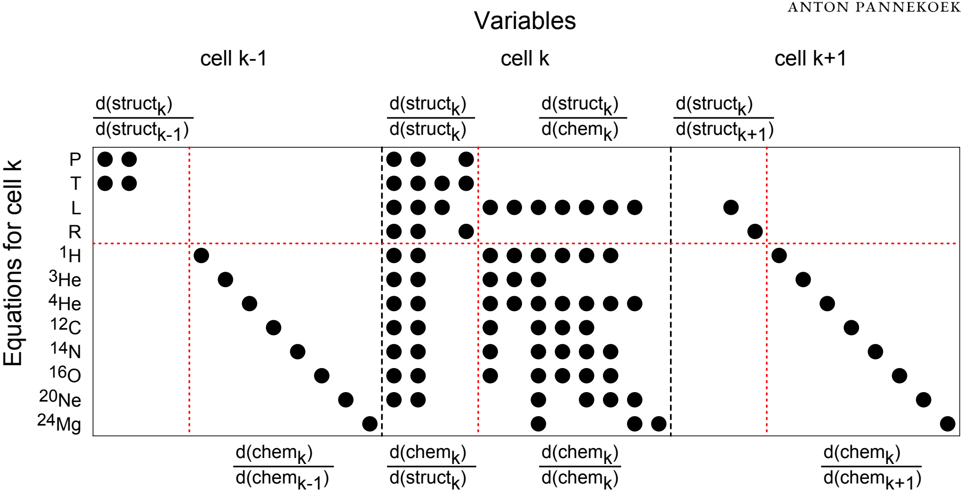

Figure: From Paxton *et al.* 2013, ApJs, 208, 4. Black dots are non-zero entries.

Numerical Methods: Algorithm

- MESA solves simultaneously the fully coupled set for the structure and composition;
- Henyey code: varies all the quantities in each zone until an acceptable solution is found ($\neq$ Shooting Method);
- Generalized Newton-Raphson solver ($\Rightarrow$ FIRST ORDER):

$$0 = \mathbb{F}(y) \simeq \mathbb{F}(y_i + \delta y_i) = \mathbb{F}(y_i) + \left[\frac{d\mathbb{F}(y)}{dy}\right]_i \delta y_i + O((\delta y_i)^2) \quad ;$$

$$\delta y_i \simeq -\frac{\mathbb{F}(y_i)}{\left[\frac{d\mathbb{F}(y)}{dy}\right]_i}$$

$$\Downarrow$$

$$y_{i+1} = y_i + \delta y_i$$

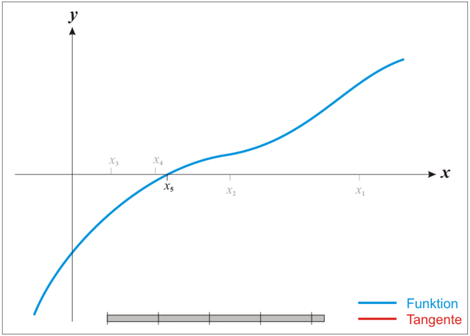

Figure: From Wikipedia

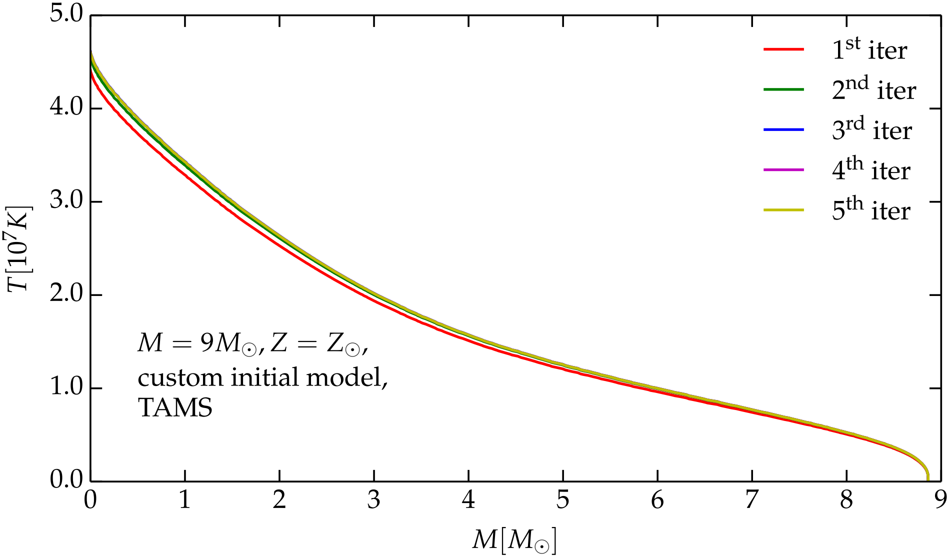

Figure: Two models after the end of core hydrogen burning

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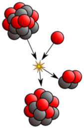
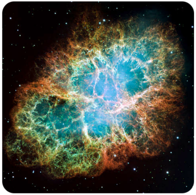
What do I do with it?

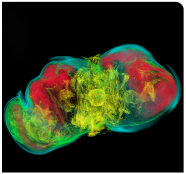
What do I do with MESA? Massive Stars

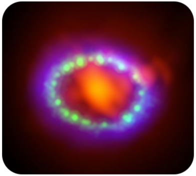
$M_{\rm ZAMS} \gtrsim 8 - 10\, M_\odot$

- Nucleosynthesis
- Chemical Evolution of Galaxies
- Effects on Star Formation
- Re-ionization Epoch
- Observations of Farthest Galaxies
- Catastrophic Events









Wind Mass Loss in Evolutionary Codes

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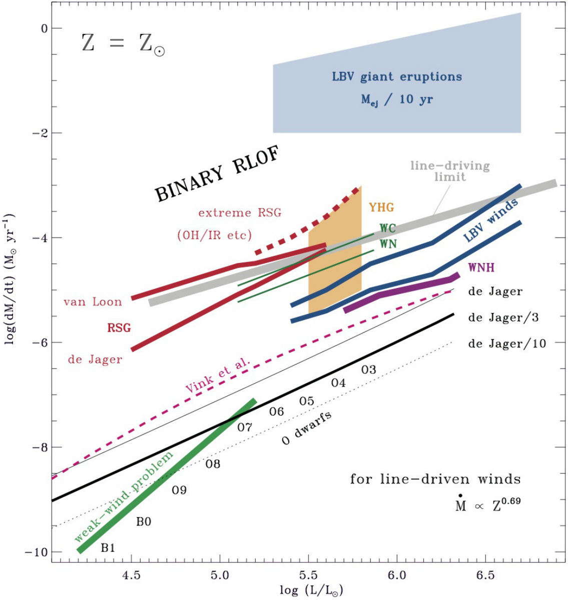

Figure: From Smith 2014, ARA&A, 52, 487S

(Semi–)Empirical parametric models. Uncertainties encapsulated in efficiency factor:

$$\dot{M}(L, T_{\rm eff}, Z, R, M, \ldots)$$

$$\Downarrow$$

$$\eta \dot{M}(L, T_{\rm eff}, Z, R, M, \ldots)$$

η is a **free** parameter:

$$\eta \in [0, +\infty)$$

My current problem:

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- I Want to see small effects $\Rightarrow$ need high spatial resolution ($\Leftrightarrow$ also high temporal resolution);
- I want to see them right-before the SN-explosion $\Rightarrow$ need to deal with advanced burning stages $\Rightarrow$ Need Large Nuclear Reaction Network;

Typically:

cells $N_Z \sim 10^5 - 10^6 \Rightarrow \mathcal{N} \simeq N_z \times N_{\rm iso} \sim 10^8 \Rightarrow$
isotopes $N_{\rm iso} \geq 200$
64bit float $\sim$ 8 bytes

$\mathcal{N}^2 \times (8\,\text{bytes}) \sim 10^{17}\,\text{bytes} \sim 10^8\,\text{Gb}$!!

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How can solve it?

Lower N_z &


Fig: Cartesius

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Fig: Cartesius

Thank you for your attention!

Numerical Methods: Timestep selection

To choose the next timestep Δt_{n+1}:

1. $v_c \leq v_t \sim 10^{-4}$, v_c unweighted average over all cells of the relative variations of $\log_{10}(R)$, $\log_{10}(T)$, $\log_{10}(\rho)$:

$$\Delta t_{n+1} = \Delta t_n \times g\left(\frac{g(v_t/v_{c,n})g(v_t/v_{c,n-1})}{g(\Delta t_n/\Delta t_{n-1})}\right)^{1/4}$$

$$g(x) \stackrel{\text{def}}{=} 1 + 2\tan^{-1}(0.5(x-1)) \quad ;$$

2. extra controls on relative variations of many quantities ($X_{i,k}$, $\varepsilon_{\mathrm{nuc},k}$, L_k, T_{eff}, etc.);

It is always possible that you need to reduce Δt_n

If MESA fails: first `retry` then `backup`

◂ Back