Day 1

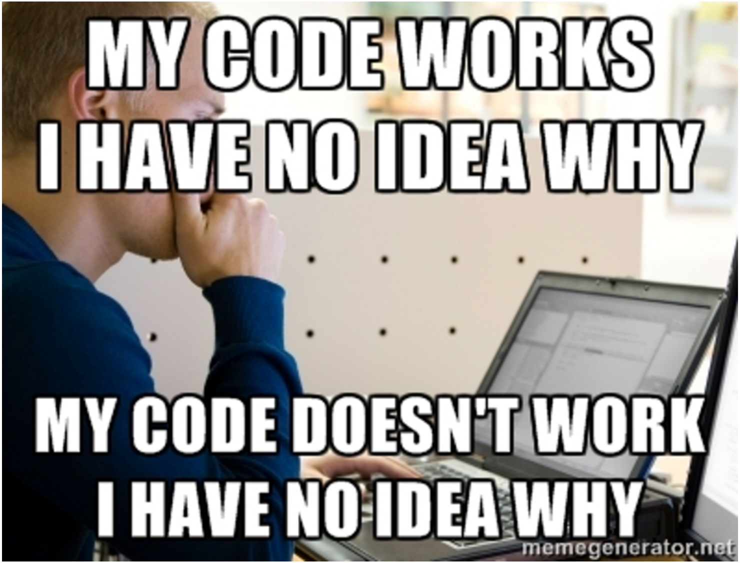
MESA

Mathieu Renzo - Università di Pisa

Summary

- Introduction
- Numerical Methods
  - 1D (or 1.5D): Spherical Symmetry
  - Meshing
  - Timestep control
  - Reformulation of the equations
  - Nuclear Networks
  - Algorithm
- Solver efficiency and run time
- How to use MESA
- Massive stars: Sensitivity to "code physics"
  - Super Eddington envelopes
  - Tweak MESA with `run_star_extras.f`: wind mass loss scheme.

Foreword


MY CODE WORKS
I HAVE NO IDEA WHY
MY CODE DOESN'T WORK
I HAVE NO IDEA WHY
memegenerator.net

Intoduction: Open Source stellar evolution code

What does it stand for?

Modules for
Experiments in
Stellar
Astrophysics

References

Paxton *et al.* 2011, ApJs192,3

Paxton *et al.* 2013, ApJs208,4

mesa.sourceforge.net

mesastar.org

Open Source ⇔ Open Know How

written in FORTRAN 90

Modules overview

MESA Module Definitions and Purposes

Name	Type	Purpose
alert	Utility	Error handling
atm	Microphysics	Gray and non-gray atmospheres; tables and integration
const	Utility	Numerical and physical constants
chem	Microphysics	Properties of elements and isotopes
diffusion	Macrophysics	Gravitational settling and chemical and thermal diffusion
eos	Microphysics	Equation of state
interp_1d	Numerics	One-dimensional interpolation routines
interp_2d	Numerics	Two-dimensional interpolation routines
ionization	Microphysics	Average ionic charges for diffusion
jina	Macrophysics	Large nuclear reaction nets using reaclib
kap	Microphysics	Opacities
karo	Microphysics	Alternative low-T opacities for C and N enhanced material
mlt	Macrophysics	Mixing length theory
mtx	Numerics	Linear algebra matrix solvers
net	Macrophysics	Small nuclear reaction nets optimized for performance
neu	Microphysics	Thermal neutrino rates
num	Numerics	Solvers for ordinary differential and differential-algebraic equations
package_template	Utility	Template for creating a new MESA module
rates	Microphysics	Nuclear reaction rates
screen	Microphysics	Nuclear reaction screening
star	Evolution	One-dimensional stellar evolution
utils	Utility	Miscellaneous utilities
weaklib	Microphysics	Rates for weak nuclear reactions

Figure: From Paxton *et al.* 2011, ApJs, 192, 3

Each module: "public" interface and "private" implementation

Numerical Methods: 1D (or 1.5D)

Prohibitive computational cost of 3D simulations

$\Rightarrow$ 1D, but stars are not spherical-symmetric!

Need of parametric approximations for:

- Rotation $\Rightarrow$ "Shellular Approximation"
- Magnetic Fields
- Convection $\Rightarrow$ Mixing Length Theory (MLT)
- ...

Beware of systematic errors!

... but stars are not necessarily static!

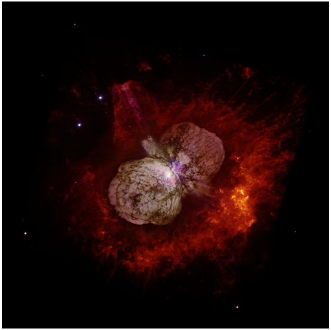


Figure: η Car, false colors, from wikipedia.

Other examples: He flash, outburst, episodic mass loss, RLO, etc...

For numerical solutions:

$$\frac{df}{dm} \rightarrow \frac{f(m_{k+1}) - f(m_k)}{m_{k+1} - m_k}$$

$\Rightarrow$ Discretization of space (mesh or grid) and time (timesteps)

Numerical Methods: Meshing

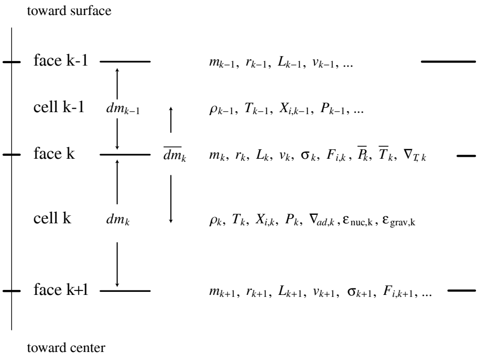


Figure: From Paxton *et al.* 2011, ApJs, 192, 3

`mesh_delta_coeff`(=1.0) , `mesh_delta_coeff_for_highT`(=1.5)

Numerical Methods: Timestep selection

To choose the next timestep Δt_{k+1}:

1. $v_c \leq v_t \sim 10^{-4}$, v_c unweighted average over all cells of the relative variations of $\log(R)$, $\log(T)$, $\log(\rho)$:

$$\Delta t_{k+1} = \Delta t_k g \left(\frac{g(v_t/v_{c,k}) g(v_t/v_{c,k-1})}{g(\Delta t_k/\Delta t_{k-1})} \right)^{1/4}$$

$$g(x) \stackrel{\text{def}}{=} 1 + 2\tan^{-1}(0.5(x-1)) \quad ;$$

2. extra controls on many quantities to reduce it if necessary;

- **Coming soon:** check that $\Delta t_{k+1} \leq \tau_{\mathrm{KH}}, \tau_{\mathrm{M\ loss}}, \tau_{\mathrm{ff}}, \ldots$

If MESA fails: first `retry` then `backup`

Numerical Methods: Reformulation of the equations

- $\frac{P_{k-1}-P_k}{0.5(dm_{k-1}-dm_k)} = -\frac{Gm_k}{4\pi r_k^4} - \frac{a_k}{4\pi r_k^2}, a_k \stackrel{\text{def}}{=} \frac{dv_k}{dt}, v_k = r_k \frac{d\ln(r_k)}{dt}$
- $\ln(r_k) = \frac{1}{3}\ln\left[r_{k+1}^3 + \frac{3}{4\pi}\frac{dm_k}{\rho_k}\right]$,
- $\frac{T_{k-1}-T_k}{0.5(dm_{k-1}-dm_k)} = -\nabla_{T,k}\left(\left.\frac{dP}{dm}\right|_k\right)_{\text{static}} \frac{<T_k>}{<P_k>}$,
- $L_k - L_{k+1} = dm_k\{\varepsilon_{\text{nuc}} - \varepsilon_\nu + \varepsilon_{\text{grav}}\}$,
- $X_{i,k}(t+\delta t) = X_{i,k}(t) + \delta t\left(\frac{dX_{i,k}}{dt}\right)_{\text{nuc}} + \frac{(X_{i,k}-X_{i,k-1})\sigma_k\delta t}{0.5(dm_{k-1}-dm_k)}$

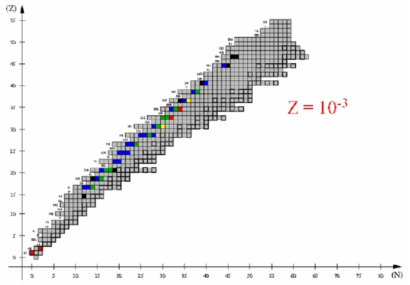


Figure: From José & Iliadis 2011, RRPh, 74, 6901

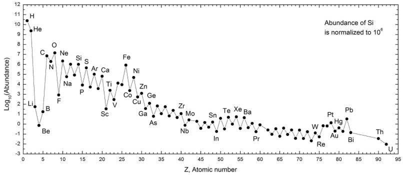


Figure: From faculty.buffalostate.edu

Small network $\Rightarrow$ fake e^--captures $\Rightarrow$ "wrong" $Y_e \stackrel{\text{def}}{=} \sum_i X_i \frac{Z_i}{A_i}$

Numerical Methods: The Matrix to Solve

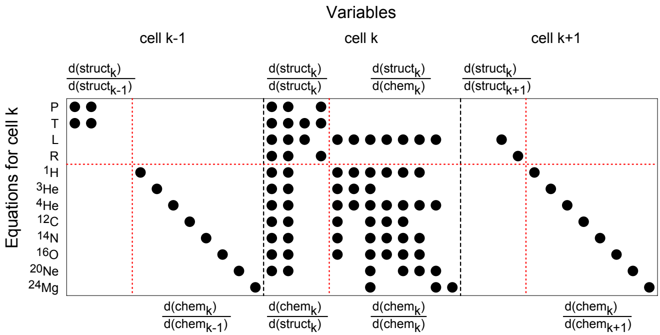


Figure: From Paxton *et al.* 2013, ApJs, 208, 4. Black dots are non-zero entries.

Numerical Methods: Algorithm

- Solves simultaneously the fully coupled set for the structure and composition;
- Henyey code: varies all the quantities in each zone until an acceptable solution is found ($\neq$ Shooting Method);
- Generalized Newton-Raphson solver ($\Rightarrow$ FIRST ORDER):

$$0 = \mathbb{F}(y) = \mathbb{F}(y_i + \delta y_i) = \mathbb{F}(y_i) + \left[\frac{d\mathbb{F}(y)}{dy}\right]_i \delta y_i + O((\delta y_i)^2)$$

$$\delta y_i \simeq -\frac{\mathbb{F}(y_i)}{\left[\frac{d\mathbb{F}(y)}{dy}\right]_i} \Rightarrow y_{i+1} = y_i + \delta y_i$$

Numerical Methods: Convergence Low M

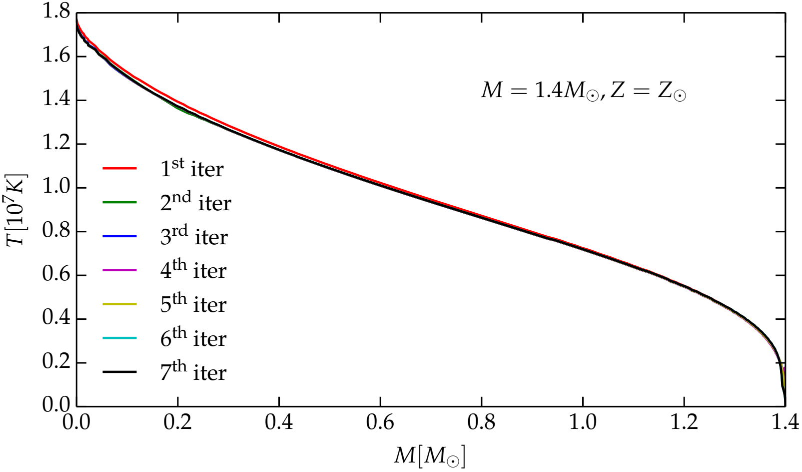


Figure: Two models after ZAMS

Numerical Methods: Convergence Intermediate M

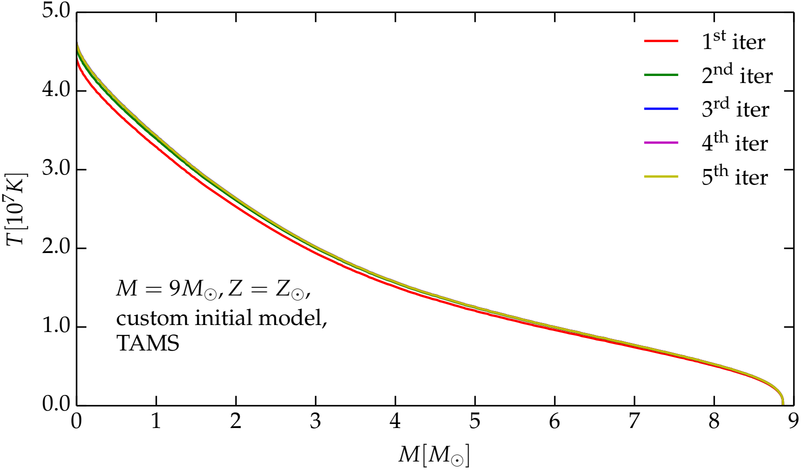


Figure: Two models after TAMS

Numerical Methods: Convergence High M

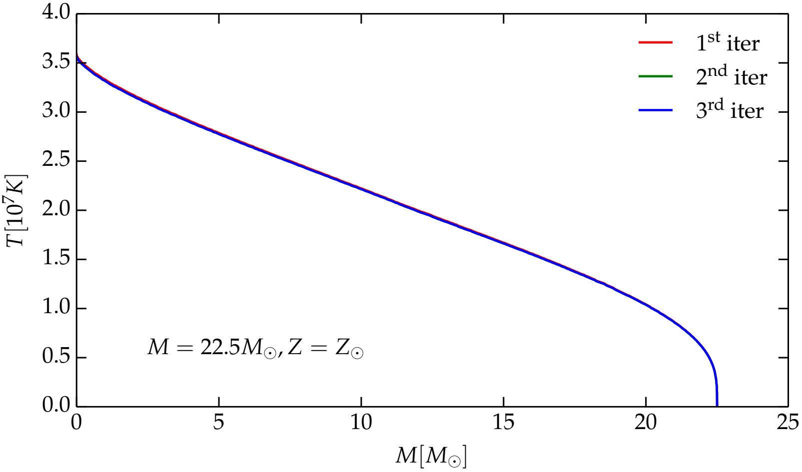


Figure: Two models after ZAMS

Numerical Methods: Run Time

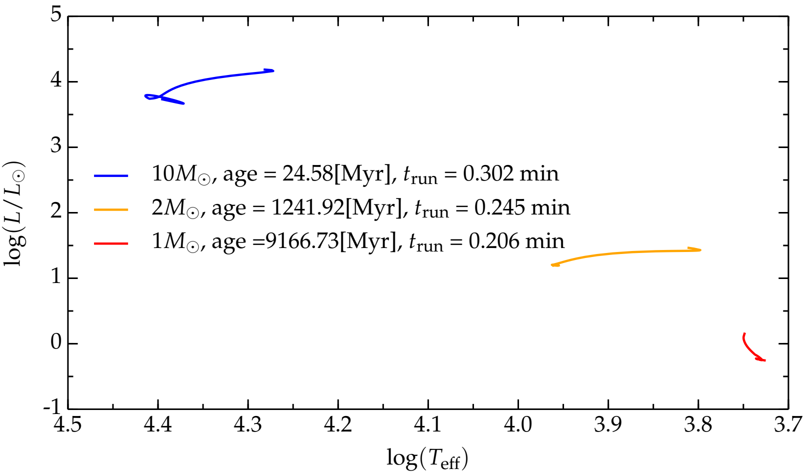


Figure: MS evolutionary tracks for 3 solar metallicity stars

Let's go down the rabbit hole...

- $MESA_DIR/star
- Work directory structure
  - output: photos, models, profiles*.data and history.data
  - compile and run
  - set up your run: the inlists
- Give it a try: will a 10 $M_\odot$ star explode?[See §4 of the notes]

Welcome to the rabbit hole!

```
ssh -YA mesawork@astr4pi.df.unipi.it
```

or

```
ssh -YA mesawork@astr18pi.df.unipi.it
```

then

```
cd ./MESA_WORKSHOP
```

Day 2

MESA

Mathieu Renzo - Università di Pisa

Summary

- How to use MESA
- Massive stars: Sensitivity to "code physics"
  - Super Eddington envelopes
  - Mass loss problem
  - How to use `run_star_extras.f`: implement a custom wind scheme

Stellar evolution calculations rely on many parametric models
(e.g. MLT, diffusion approximation, rotation, mass loss, B-fields, etc...)
and poorly constrained physical quantities
(e.g. $^{12}C(\alpha,\gamma)^{16}O$ rate)
Their influence is not always explored in a systematic way.

Super Eddington Envelopes 1

$$\frac{dP_{\rm rad}}{dr}(r) \stackrel{!}{=} -\frac{GM(r)\rho(r)}{r^2} \Rightarrow L_{\rm Edd}(r) \stackrel{\rm def}{=} \frac{4\pi GM(r)c}{\kappa(r)}$$

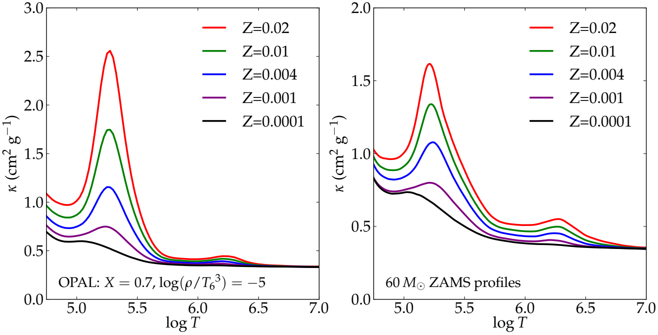


Figure: From Paxton *et al.* 2013, ApJs, 208, 4

Super Eddington Envelopes 2

$$\text{Convection} \Leftrightarrow \frac{dP_{\rm rad}}{dP_{\rm tot}} \geq \left.\frac{\partial P_{\rm rad}}{\partial P_{\rm tot}}\right|_s, \text{ but } F_{\rm conv} \lesssim \rho c_s^3$$

$$\begin{cases} L(r) > L_{\rm Edd}(r) \\ F \gtrsim F_{\rm rad} + \rho c_s^3 \end{cases} \Rightarrow$$

MLT++:

`okay_to_reduce_gradT`

$$\nabla_T - \nabla_{\rm ad} \to \alpha_\nabla f_\nabla (\nabla_T - \nabla_{\rm ad})$$

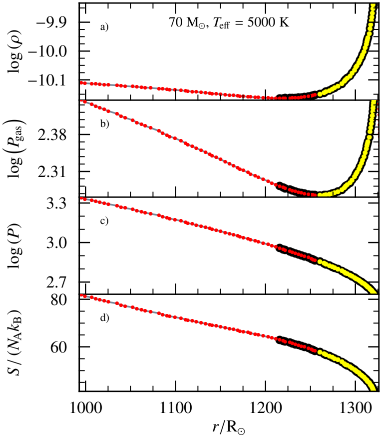


Figure: From Paxton *et al.* 2013

Effects of MLT++

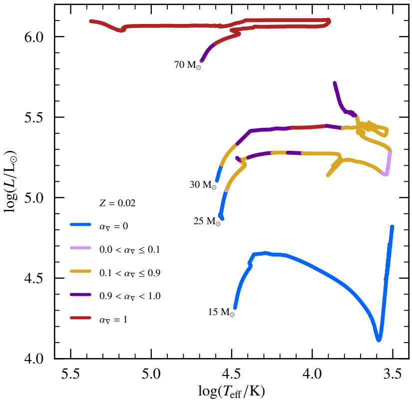


Figure: From Paxton *et al.* 2013, ApJs, 192, 3

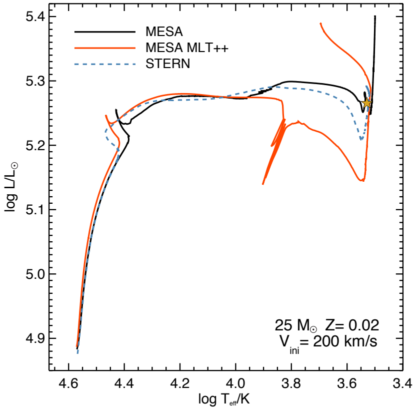


Figure: From Paxton *et al.* 2013, ApJs, 192, 3

Massive stars mass loss in one slide

Important for:

- Chemical and dynamical evolution of galaxies
- Evolutionary time-scales
- Final remnant (NS or BH)

Mechanisms:

- Line-driven winds
- Binary interactions
- Eruption and episodic events
- ...

How to simulate it:

Assumption of steadiness and homogeneity $\Rightarrow$ Time averaged prescriptions,

$$\dot{M} \equiv \dot{M}(L, T_{\rm eff}, R, M, Z, ...)$$

Large uncertainties encapsulated in fudge factors

$$\dot{M} \to \eta \dot{M}$$

Mass Loss Rates

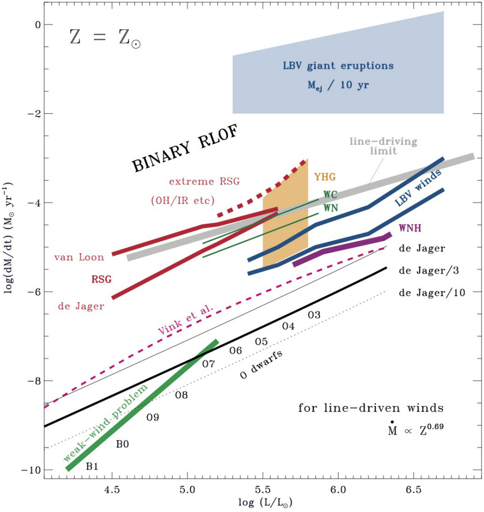


Figure: From Smith 2014, ARA&A, 52, 487S

MESA's mass loss recipies

- RSG_wind_scheme
  used if $X_c > 0.01$ or $Y_c >$ RGB_to_AGB_wind_switch
- AGB_wind_scheme
  used if $X_c < 0.01$ and $Y_c <$ RGB_to_AGB_wind_switch

In (`$MESA_DIR/star/defaults/controls.default`): Reimers, Blocker, de Jager, van Loon, Nieuwenhuijzen, Kudritzki, Vink, Dutch or

`other`⇒ allows the implementation of a custom mass loss prescription using the `run_star_extras.f` and the `other` `‘‘hooks’’`.