

H_{mhf}:

$$\begin{aligned}
\langle N'\Lambda SJ'F | H_{mhf} | N\Lambda SJF \rangle &= a\Lambda(-1)^{2N'+2J+I+F+S-\Lambda+1} \\
&\times \sqrt{(2J+1)(2J'+1)(2N+1)(2N'+1)I(I+1)(2I+1)} \\
&\times \begin{pmatrix} N' & 1 & N \\ -\Lambda & 0 & \Lambda \end{pmatrix} \begin{Bmatrix} J' & I & F \\ I & J & 1 \end{Bmatrix} \begin{Bmatrix} N' & J' & S \\ J & N & 1 \end{Bmatrix} \\
&+ b_F \delta_{NN'} (-1)^{N+J+J'+I+F+S+1} \\
&\times \sqrt{(2J+1)(2J'+1)I(I+1)(2I+1)S(S+1)(2S+1)} \\
&\times \begin{Bmatrix} J' & I & F \\ I & J & 1 \end{Bmatrix} \begin{Bmatrix} S & J' & N \\ J & S & 1 \end{Bmatrix} \\
&+ b_{FD} \delta_{NN'} N(N+1) (-1)^{N+J+J'+I+F+S+1} \\
&\times \sqrt{(2J+1)(2J'+1)I(I+1)(2I+1)S(S+1)(2S+1)} \\
&\times \begin{Bmatrix} J' & I & F \\ I & J & 1 \end{Bmatrix} \begin{Bmatrix} S & J' & N \\ J & S & 1 \end{Bmatrix} \\
&+ \frac{c\sqrt{10}}{\sqrt{3}} (-1)^{J+I+F+N'-\Lambda+1} \\
&\times \sqrt{(2N+1)(2N'+1)(2J+1)(2J'+1)I(I+1)(2I+1)S(S+1)(2S+1)} \\
&\times \begin{pmatrix} N' & 2 & N \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} J' & I & F \\ I & J & 1 \end{Bmatrix} \begin{Bmatrix} N' & N & 2 \\ S & S & 1 \\ J' & J & 1 \end{Bmatrix} \\
&+ C_I \delta_{NN'} (-1)^{2J+I+F+N'+S+1} \\
&\times \sqrt{N(N+1)(2N+1)(2J+1)(2J'+1)I(I+1)(2I+1)} \\
&\times \begin{Bmatrix} N' & J' & S \\ J & N & 1 \end{Bmatrix} \begin{Bmatrix} J' & I & F \\ I & J & 1 \end{Bmatrix}
\end{aligned}$$

For $S > 1$ *:

$$\begin{aligned} \langle N'SJ'F | H_{mf}^{(3)} | NSJF \rangle &= \frac{b_s \sqrt{35}}{4\sqrt{3}} (-1)^{N+J'+I+F} \\ &\times \sqrt{(2N+1)(2N'+1)(2J+1)(2J'+1)I(I+1)(2I+1)} \\ &\times \begin{pmatrix} N & 2 & N' \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} F & J & I \\ 1 & I & J' \end{Bmatrix} \begin{Bmatrix} N & N' & 2 \\ S & S & 3 \\ J & J' & 1 \end{Bmatrix} \prod_{k=0}^6 \sqrt{2S+k-2} \end{aligned}$$

* A.S-C. Cheung and A.J. Merer, *Molecular Physics*, **46** (1), 111-128 (1982).