## $\ell$ - type doubling:

For $v_{degenerate}=1^1$:

$$\langle N\Lambda SJ, v=1, l=\pm 1 | H_{l-doub} | N\Lambda SJ, v=1, l=\pm 1\rangle = \pm\frac{q}{2}J(J+1) \mp \frac{q_D}{2}J^2(J+1)^2 \pm \frac{q_H}{2}J^3(J+1)^3$$

For the v = 2, 3, and $4^2$:

$E_l^0$ represents the origin of each l-component.

$$\langle N\Lambda SJ, v=2 | H_{l-doub} | N\Lambda SJ, v=2\rangle \doteq \begin{pmatrix} E_2^0 & W_{20} & W_{22} \\ W_{20} & E_0^0 & W_{20} \\ W_{22} & W_{20} & E_2^0 \end{pmatrix}$$

$$W_{20} = \frac{q}{\sqrt{2}}\sqrt{J^2(J+1)^2 - 2J(J+1)} \qquad W_{22} = \frac{\rho}{2}\sqrt{J^2(J+1)^2 - 2J(J+1)}$$

$$\langle N\Lambda SJ, v=3 | H_{l-doub} | N\Lambda SJ, v=3\rangle \doteq \begin{pmatrix} E_3^0 & W_{31} & W_{3,-1} & 0 \\ W_{31} & E_1^0 & W_{11} & W_{3,-1} \\ W_{3,-1} & W_{11} & E_1^0 & W_{31} \\ 0 & W_{3,-1} & W_{31} & E_3^0 \end{pmatrix}$$

$$W_{31} = \frac{\sqrt{3}q}{2}\sqrt{J^2(J+1)^2 - 8J(J+1) + 12}$$

$$W_{3,-1} = \frac{\sqrt{3}\rho}{2}J(J+1)\sqrt{J^2(J+1)^2 - 8J(J+1) + 12} \qquad W_{11} = qJ(J+1)$$

For v = 4, Maki and Lide[3] list two matrices to be diagonalized simultaneously.

$$H_{l-doubling:v=4} \doteq \begin{pmatrix} E_\Gamma^0 & W_{42} & \sqrt{2}W_{40} \\ W_{42} & E_\Delta^0 & \sqrt{2}W_{02} \\ \sqrt{2}W_{40} & \sqrt{2}W_{02} & E_\Sigma^0 \end{pmatrix} \quad \text{and} \quad H_{l-doubling:v=4} \doteq \begin{pmatrix} E_\Delta^0 - W_{2,-2} & W_{42} \\ W_{42} & E_\Gamma^0 \end{pmatrix}$$

$$W_{42} = q\sqrt{J^2(J+1)^2 - 18J(J+1) + 72}$$

$$W_{40} = \frac{\sqrt{6}}{2}\rho\sqrt{\left(J^2(J+1)^2 - 18J(J+1) + 72\right)\left(J^2(J+1)^2 - 2J(J+1)\right)}$$

$$W_{02} = \frac{\sqrt{6}}{2}q\sqrt{J^2(J+1)^2 - 2J(J+1)} \qquad W_{2,-2} = \frac{3}{2}\rho J(J+1)\left[J(J+1) - 2\right]$$


1. P.I. Presunka and J.A. Coxon, *Can. J. Chem.*, **71**, 1689 (1993).
2. Yamada, K.M.T., Birss, F.W., and Aliev, M.R., *J. Mol. Spec.*, **112** 347 (1985).
3. A.G. Maki and D.R. Lide, Jr., *J. Chem. Phys.*, **47** (9), 3206 (1967).