

## Lambda doubling in $\Pi$ states:

$$\begin{aligned}
\langle N' \Lambda S J \pi | H_{ld} | N \Lambda S J \pi \rangle &= p \pi \sqrt{5} (-1)^{N+S+J+\Lambda+1} \sqrt{S(S+1)(2S+1)(2N'+1)N(N+1)} \\
&\times (2N+1) \begin{pmatrix} N' & 2 & N \\ -\Lambda & 2 & \Lambda \end{pmatrix} \begin{Bmatrix} N & S & J \\ S & N' & 1 \end{Bmatrix} \begin{Bmatrix} 1 & N & N \\ N' & 2 & 1 \end{Bmatrix} \\
&+ p_D (N(N+1) + N'(N'+1)) \pi \frac{\sqrt{5}}{2} (-1)^{N+S+J+\Lambda+1} \\
&\times \sqrt{S(S+1)(2S+1)(2N'+1)N(N+1)} (2N+1) \begin{pmatrix} N' & 2 & N \\ -\Lambda & 2 & \Lambda \end{pmatrix} \\
&\times \begin{Bmatrix} N & S & J \\ S & N' & 1 \end{Bmatrix} \begin{Bmatrix} 1 & N & N \\ N' & 2 & 1 \end{Bmatrix} \\
&+ \frac{q}{2\sqrt{6}} \pi \delta_{NN'} (-1)^{N'+N-\Lambda} \begin{pmatrix} N' & 2 & N \\ -\Lambda & 2 & -\Lambda \end{pmatrix} \prod_{k=0}^4 \sqrt{2N+k-1} \\
&+ \frac{q_D}{2\sqrt{6}} \pi \delta_{NN'} (-1)^{N'+N-\Lambda} N(N+1) \begin{pmatrix} N' & 2 & N \\ -\Lambda & 2 & -\Lambda \end{pmatrix} \prod_{k=0}^4 \sqrt{2N+k-1}
\end{aligned}$$

For  $S > 1/2$  include:

$$\begin{aligned}
\langle N' \Lambda S J \pi | H_{ld} | N \Lambda S J \pi \rangle &= \frac{o}{2\sqrt{6}} \pi (-1)^{2N+S+J+N'-\Lambda} \sqrt{(2N'+1)(2N+1)} \prod_{k=0}^4 \sqrt{2S+k-1} \\
&\times \begin{pmatrix} N' & 2 & N \\ -\Lambda & 2 & \Lambda \end{pmatrix} \begin{Bmatrix} S & N' & J \\ N & S & 2 \end{Bmatrix} \\
&+ \frac{o_D}{4\sqrt{6}} (N(N+1) + N'(N'+1)) \pi (-1)^{2N+S+J+N'-\Lambda} \\
&\times \sqrt{(2N'+1)(2N+1)} \prod_{k=0}^4 \sqrt{2S+k-1} \begin{pmatrix} N' & 2 & N \\ -\Lambda & 2 & \Lambda \end{pmatrix} \begin{Bmatrix} S & N' & J \\ N & S & 2 \end{Bmatrix}
\end{aligned}$$