

$$H_{\text{so}}^{(3)} = +\eta L_z S_z \left[S_z^2 - \frac{(3S^2 - 1)}{5} \right] + \frac{\eta_D}{2} \left\{ L_z S_z \left[S_z^2 - \frac{(3S^2 - 1)}{5} \right], (J - S)^2 \right\}$$