

$$H_{\text{mhf}} = aI_z L_z + b(\mathbf{I} \cdot \mathbf{S}) + cI_z S_z - \frac{d}{2} (S_+ I_+ e^{-2i\phi} + S_- I_- e^{+2i\phi})$$