

$$\begin{aligned}
H_{LD,\Delta} &= \frac{1}{2} \tilde{m}_\Delta (\mathbf{S}_+^4 + \mathbf{S}_-^4) - \frac{1}{2} \tilde{n}_\Delta (\mathbf{S}_+^3 \mathbf{J}_+ + \mathbf{S}_-^3 \mathbf{J}_-) \\
&+ \frac{1}{2} \tilde{o}_\Delta (\mathbf{S}_+^2 \mathbf{J}_+^2 + \mathbf{S}_-^2 \mathbf{J}_-^2) - \frac{1}{2} \tilde{p}_\Delta (\mathbf{S}_+ \mathbf{J}_+^3 + \mathbf{S}_- \mathbf{J}_-^3) \\
&+ \frac{1}{2} \tilde{q}_\Delta (\mathbf{J}_+^4 + \mathbf{J}_-^4)
\end{aligned}$$