

$$\begin{aligned}
\mathbf{H}_{\Lambda\text{-doubling}} &= \frac{1}{2}(\mathbf{S}_+^2 + \mathbf{S}_-^2) - \frac{1}{2}(\mathbf{p} + 2\mathbf{q})(\mathbf{J}_+\mathbf{S}_+ + \mathbf{J}_-\mathbf{S}_-) + \frac{1}{2}\mathbf{q}(\mathbf{J}_+^2 + \mathbf{J}_-^2) \\
&\quad + \frac{1}{4}\{(\mathbf{S}_+^2 + \mathbf{S}_-^2), (\mathbf{J} - \mathbf{S})^2\} \\
&\quad - \frac{1}{4}(\mathbf{p}_D + 2\mathbf{q}_D)\{(\mathbf{J}_+\mathbf{S}_+ + \mathbf{J}_-\mathbf{S}_-), (\mathbf{J} - \mathbf{S})^2\} + \frac{1}{4}\mathbf{q}_D\{(\mathbf{J}_+^2 + \mathbf{J}_-^2), (\mathbf{J} - \mathbf{S})^2\}
\end{aligned}$$