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| **Introduction** |

**Prerequisite Knowledge**

* Basic familiarity with the curvature of spacetime caused by the presence of mass
* Basic familiarity with how spacetime curvature affects the path traveled by light
* Basic familiarity with brightness vs. time graphs

**Goals**

* Understand how gravitational microlensing causes changes in the apparent brightness of distant light sources.
* Understand how the mass and alignment of the lensing object(s) affect the brightness vs. time curve of the light source.
* From a graph of brightness vs. time for a light source affected by gravitational microlensing, infer information about the objects causing the lensing effect.
* From a description of a star-exoplanet system causing a microlensing effect, produce qualitatively correct graphs of the light source’s brightness vs. time

**Pre-activity Questions**

1. Which graph below best represents the observed brightness of a distant background star over time as a nearby star moves between the distant star and Earth?

Brightness

Brightness

A

**B**

Time

Time

Brightness

Brightness

D

C

Time

Time

1. A star-exoplanet system moves from left to right as seen from Earth. The exoplanet is to the right of the star when the system crosses between Earth and a background object. Which of the following statements best describes the brightness vs. time graph for the background object?
2. **The bump caused by the planet will occur earlier than the peak caused by the star.**
3. The bump caused by the planet will occur later than the peak caused by the star.
4. The bump caused by the planet will occur at the same time as the peak caused by the star.

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| **Tutorial Guide** |

**Figure: Cases 1 to 3**

These three cases provide a set of graphical representations for how light from a distant star (Star S) may be affected by matter in the region between Star S and Earth. In each case, light from Star S is bent by a different amount as the result of spacetime curvature in the region.

1. **[ 3 > 2 > 1 ]**
2. **[ 3 > 2 > 1 ]** Light from Star S is bent more as a result of a greater warping of spacetime in the region between Star S and Earth by the lensing object. Case 3 shows the most light from Star S being bent, Case 2 shows less light from Star S being bent, and Case 1 shows no light from Star S being bent. This means that the lensing object in Case 3 warps spacetime the greatest, in Case 2 the lensing object warps spacetime less than Case 3, and in Case 1 there is no lensing object present, so the spacetime is not warped.
3. **[ 3 > 2 > 1 ]** Spacetime is more warped when more mass is present. More warping of spacetime in the region between Star S and Earth will cause light from Star S to be bent more. Case 3 shows the most light from Star S being bent, Case 2 shows less light from Star S being bent, and Case 1 shows no light from Star S being bent. This means that the lensing object in Case 3 is the most massive, in Case 2 the lensing object is less massive, and in Case 1 there is no lensing object present.

**Figure: Cases 4 to 6**

This set of three cases explicitly shows the curvature of spacetime between Star S and Earth. Light from Star S is not shown.

1. **[ Cases 4 & 2; Cases 5 & 1; Cases 6 & 3 ]** In Case 2, the light is bent somewhat, so there must be some warping of spacetime, like in Case 4. In Case 1, the light from Star S is not bent, so the region of spacetime is flat – this matches the picture in Case 5. Finally, in Case 3 the light is bent a great amount, so the spacetime must be very warped, which matches Case 6.
2. **[ Lensing object for Case 6 ]** The region of spacetime warped by the lensing object in Case 6 will take longer to cross through our field of view. Since both lensing objects are moving at the same speed, but the region of warped spacetime for Case 6 is larger, it will take longer for the region of spacetime for Case 6 to move across our field of view.

*Struggling students may have missed that the lensing objects are traveling at the same speed. First, ask students to read the introduction to Question 5. Students continuing to struggle may not be connecting that the lensing objects are moving at the same speed with the lensing object in Case 6 warping a larger region of spacetime, as well as how this would affect the travel time of the warped regions of spacetime. Ask students “Are the lensing objects traveling at the same speed or different speeds?”, then “Are the lensing objects warping the same sized region of spacetime or different?”, and finally “Which lensing object is bigger?” At this point, you can have students connect the pieces by asking, “If both lensing objects are moving at the same speed, and the higher mass lensing objects warps a larger region of spacetime, which region will take longer to move through the field of view?”*

1. **[ Graph A & Case 4; Graph B & Case 6 ]** The region of spacetime warped by the lensing object in Case 6 will take longer to cross through our field of view than the region of spacetime warped by the lensing object in Case 4. This will cause the brightness peak for Case 6 to be wider (longer duration) than the brightness peak for Case 4. The brightness peak in Graph B is wider than the brightness peak in Graph A. This means that Graph B matches with Case 6, and Graph A matches with Case 4.
2. *The lensing object in Case 4 warps spacetime \_\_\_\_\_\_\_\_ (more/****less****/equally) compared to the lensing object in Case 6 because the mass of the lensing object in Case 4 is \_\_\_\_\_\_\_ (greater/****less****/the same). So, the lensing object in Case 4 bends the light from the distant star \_\_\_\_\_\_\_ (more/****less****/the same) compared to the lensing object in Case 6. This means it takes a \_\_\_\_\_\_\_\_ (longer/****shorter****/equal) amount of time for the warped region of spacetime for Case 4 to move between Earth and the distant star. This will make the light curve for Case 4 \_\_\_\_\_\_\_\_\_ (wider/****narrower****/the same) compared to the light curve for Case 6. Therefore, Case 4 corresponds to\_\_\_\_\_\_\_ (****Graph A****/Graph B) because the brightness peak is\_\_\_\_\_\_\_ (wider/****narrower****/the same) compared to the brightness peak in\_\_\_\_\_\_\_ (Graph A/****Graph B****).*

*The purpose of Question 7 is to help students review and check their answers and reasoning for Questions 1-6. If they ask for help with Question 7, it can be very beneficial to have them refer to their answers to the previous questions so they can evaluate their own understanding and assess whether they have constructed a coherent and robust mental model for gravitational microlensing.*

**Figure 1 (Microlensing with Exoplanet)**

This figure shows a star (Star Z) orbited by an exoplanet (Planet P) at five consecutive times. The exoplanet system moves from left to right between Earth and a distant star.

1. **[ Star Z ]** Star Z causes a greater warping of spacetime around itself than Planet P does, because stars are more massive than planets.
2. **[ Planet P ]**
3. **[ Graph D ]** In Figure 1, Planet P comes between Earth and the distant star earlier in time than Star Z, so the bump on the graph caused by Planet P should also occur at an earlier time than the peak caused by Star Z. Since time increases from left to right on the horizontal (time) axis, the bump on the graph from the exoplanet should be to the left (earlier in time) compared to the peak caused by the star. This matches what Graph D shows.
4. **[ Student 2 ]** Student 2 is correct. Student 1 is confusing the location of the exoplanet with the position of the bump on the graph. Student 2 correctly understands that the position of the exoplanet determines whether the brightness bump caused by the exoplanet occurs at an earlier or later time than the brightness peak caused by the star.

*Students may have trouble with Questions 10 and/or 11, which address a very common reasoning difficulty. Students will often assume that if an exoplanet is to one side of its parent star, features of a graph indicating the exoplanet’s presence must be on that side of the graph. In this case, since Planet P is located to the right of Star Z, these students will state that the change in brightness caused by Planet P should occur on the right side of the graph, even though the horizontal axis of the graph is time. They may struggle to reconcile Student 2’s correct reasoning with their wrong answer. To help them overcome this difficulty, you can ask what the horizontal axis of the graph is (time) and then ask if the left side of the graph indicates earlier or later times. Then, asking whether Planet P crosses between Earth and the distant star earlier or later in time than Star Z should help the students to engage in the correct reasoning for these questions.*

1. a) **[ Graph F ]** Since the exoplanet has a low mass, the brightness bump caused by the exoplanet should be relatively narrow (short duration). Since the exoplanet is located to the right of the star and the system is moving from left to right, the brightness bump caused by the exoplanet should occur earlier in time than the brightness peak caused by the star. Both Graphs G and F have a brightness bump caused by an exoplanet happening earlier in time than the brightness peak caused by the star. Since Graph F has a narrower brightness bump caused by the exoplanet than Graph G, Graph F is the correct graph.

b) **[ Graph H ]** Since the exoplanet has a high mass, the brightness bump caused by the exoplanet should be relatively wide (long duration). Since the exoplanet is located to the left of the star and the system is moving from left to right, the brightness bump caused by the exoplanet should occur after the brightness peak caused by the star. Both Graphs E and H have a brightness bump caused by an exoplanet happening later in time than the brightness peak caused by the star. Since Graph H has a wider brightness bump caused by the exoplanet than Graph E, Graph H is the correct graph.

1. **[ Graph J ]** The star and exoplanet are moving together at the same speed. So, if the exoplanet is further from its star as seen from Earth, then a greater amount of time will pass between the brightness bump caused by the exoplanet and the brightness peak caused by its star. Graph J shows a greater amount of time between the bump and the peak.
2. **[ (Graph K Below) ]**

**[ (Graph L Below) ]**

*This question requires students to simultaneously incorporate four distinct physical variables (size, distance, direction of motion, and position) into their answers. While they have worked with each variable individually already, some students may struggle when confronted with addressing all four at once. To help these students, ask them about each of the four variables individually and then ask them to construct an answer based on their understanding of the individual variables.*

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| **Additional Questions** |

1. Which of the four brightness vs. time graphs below best represents the microlensing caused by a high-mass planet orbiting a low-mass star?
2. Which of the following lists correctly ranks the bodies by the amount of spacetime curvature they cause, from least to greatest?
   1. **Small planet, large planet, small star, large star**
   2. Small planet, small star, large planet, large star
   3. Large star, small star, large planet, small planet
   4. Large star, large planet, small star, small planet

1. How many of the below graphs are reasonable representations of the microlensing that is caused by the star-exoplanet system shown here?

Earth

a. None b. Only one **c. Two** d. Three e. All four

1. Which of the following graphs correctly represents the observed brightness vs. time of the background star, as a result of the microlensing caused by the extrasolar planet system shown here?

Earth